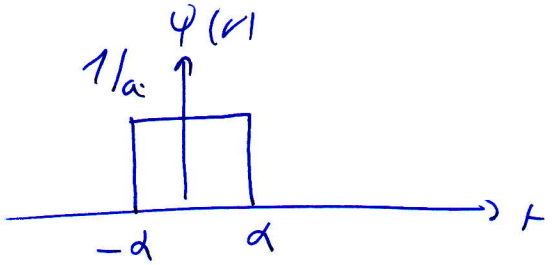
Ex 22/09/2017:

1°) $\psi(t) = \frac{1}{a} \mathbb{1}_{[-a,a]}(t)$ =>



=> signal transitoire => NRT finie.

$$E_\psi = \int_{-\infty}^{+\infty} |\psi(t)|^2 dt = \int_{-a}^{a} \frac{1}{a^2} \cdot 1^2 dt = \frac{1}{a^2}(2a) = \frac{2a}{a^2}$$

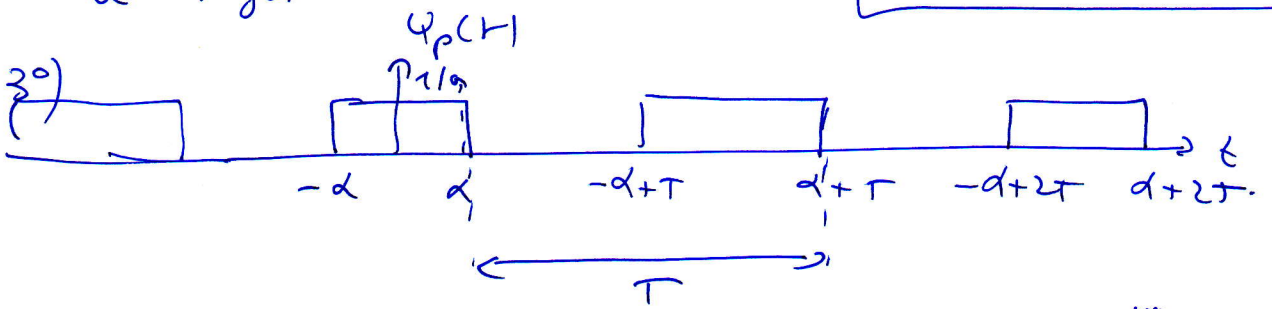
2°) Soit $\Psi(f) = TF(\psi(t)) = \int_{-\infty}^{+\infty} \frac{1}{a} \mathbb{1}_{[-a,a]}(t) e^{-2j\pi f t} dt$.

$$= \frac{1}{a} \int_{-a}^{a} e^{-2j\pi f t} dt = \frac{1}{a} \left[e^{-2j\pi f a} - e^{2j\pi f a} \right] \frac{1}{-2j\pi f}$$

$$= \frac{1}{-2ja\pi f} e^{-j\pi f a} \left(e^{-j\pi f a} - e^{j\pi f a} \right) = \frac{1}{-2ja\pi f} e^{-j\pi f a} (-2j \sin(\pi f a))$$

$$= \frac{1}{a} \frac{a}{\pi f a} \sin(\pi f a) e^{-j\pi f a} = \boxed{\frac{a}{a} \operatorname{sinc}(fa) \cdot e^{-j\pi f a}}$$

3°)



4°) $\Psi_p(f) = TF(\psi_p(t)) = TF\left(\psi(t) * \sum_{k=-\infty}^{+\infty} \delta(t - kT)\right)$

$$= TF(\psi(t)) \cdot TF\left(\sum_{k=-\infty}^{+\infty} \delta(t - kT)\right)$$

$$= \boxed{\frac{1}{T} \Psi(f) \cdot \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{T}\right) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} \Psi\left(\frac{k}{T}\right) \delta\left(f - \frac{k}{T}\right)}$$

5°) Periodiser un signal dans le domaine temporel, c'est discrétiser ou échantillonner son spectre.