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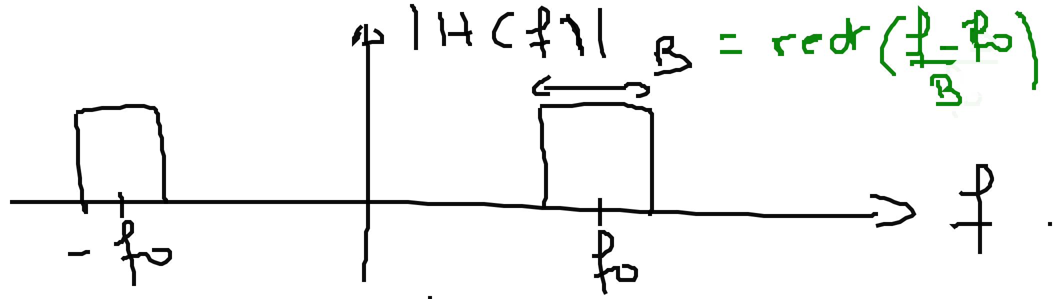
4)



$$|H(f)| = \mathrm{rect}\left(\frac{f-f_0}{B}\right) + \mathrm{rect}\left(\frac{f+f_0}{B}\right)$$

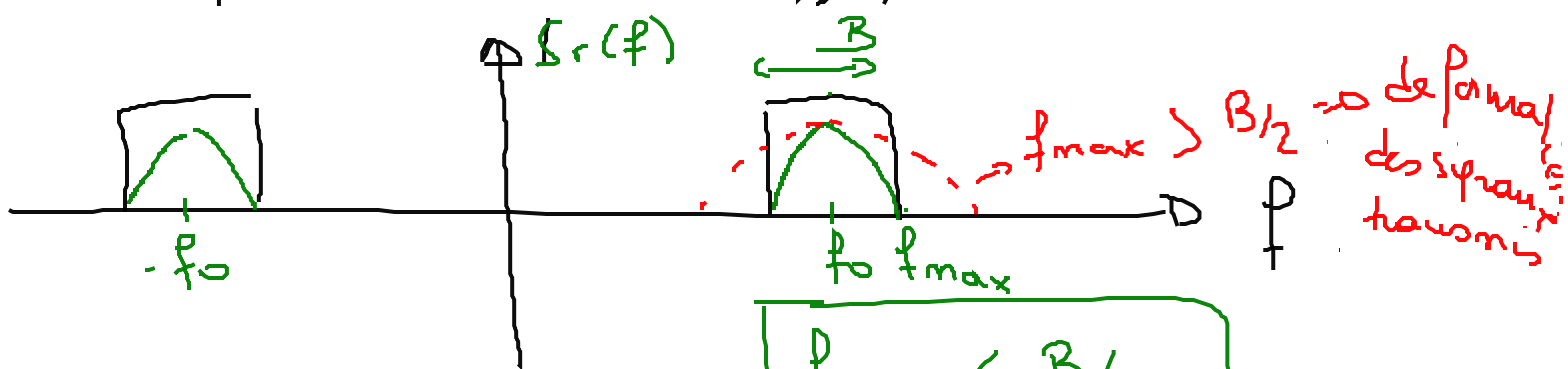
filtre passe bande.

$S_r(f)$ = DSP en sortie du filtre (de $r(t)$)

$$S_r(f) = |H(f)|^2 \cdot S_x(f)$$

$$= \frac{1}{4} S_x(f-f_0) \cdot \mathrm{rect}\left(\frac{f-f_0}{B}\right) + \frac{1}{4} S_x(f-f_0)\,\mathrm{rect}\left(\frac{f+f_0}{B}\right)$$

$$+ \frac{1}{4} \underbrace{\cancel{S_x(f-f_0) \cdot \mathrm{rect}\left(\frac{f+f_0}{B}\right)}}_{=0} + \ldots$$



$$\boxed{f_{max} < B/2}$$

5) $r(t) = x(t)$ $\quad f_{max} < B/2$.

$$r(t) = x(t) = s(t)\cos(2\pi f_0 t + \phi)$$

(on néglige w dans l'expression)

$$d(t) = r(t)\cos(2\pi f_0 t + \phi)$$

$$= s(t)\cos^2(2\pi f_0 t + \phi)$$

$\cos^2 a = \frac{1}{2}(1 + \cos 2a)$

$$= \frac{s(t)}{2} + \frac{s(t)}{2}\cos(\underbrace{4\pi f_0 t}_{2f_0 \times 2\pi t} + 2\phi)$$

Correspond à $s(t)$ le signal transmis

$s(t) \rightarrow$ centré autour de $2f_0$.

<u>Remarque</u> : Après multiplication par $\cos(2\pi f_0 t + \phi)$ $d(t)$ est filtré avec un filtre passe bas $\Rightarrow$ on récupère ainsi $\frac{s(t)}{2}$.

6) $C_d(t, t-\tau) = E\left(d(t)\, d(t-\tau)\right)$

$$= E\left(\frac{1}{2} s(t)\left(1+\cos(4\pi f_0 t + 2\phi)\right) \cdot \frac{1}{2} s(t-\tau)\left(1+\cos\left(4\pi f_0 (t-\tau) + 2\phi\right)\right)\right)$$

$$= \frac{1}{4} E\left(s(t)\, s(t-\tau)\right) \cdot E\left(\left(1+\cos(4\pi f_0 t + 2\phi)\right) \cdot \left(1+\cos(4\pi f_0(t-\tau)+2\phi)\right)\right)$$

(7.2)

$$= \frac{1}{4} C_s(\tau) \cdot \Big[E(1) + \underbrace{E\left(\cos(\cancel{4\pi f_0 t} + 2\phi)\right)}_{=0} + \underbrace{E\left(\cos(4\pi \cancel{f_0}(t-\tau)+2\phi)\right)}_{0} + \underbrace{E\left(\cos(4\pi f_0 t + 2\phi)\cos(4\pi f_0(t-\tau)+2\phi)\right)}$$

$$\boxed{= \frac{1}{4} C_s(\tau)\left(1 + \frac{1}{2}\cos 4\pi f_0 \tau\right)}$$

indépendante de t.

(*) $E(\cos a \cdot \cos b) = E\left(\frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)\right)$

$$E\left(\frac{1}{2}\cos 4\pi f_0 \tau + \underbrace{\frac{1}{2}\cos\left(4\pi f_0 (2t-\tau) + 4\phi\right)}_{=0}\right)$$

indépendant de ϕ

$$E(\cdot) = \frac{1}{2}\int_0^{2\pi} \cos\left(4\pi f_0(2t-\tau)+4\phi\right)\frac{1}{2\pi}\,d\phi = 0$$

$$S_d(f) = TF(C_d(\tau))$$

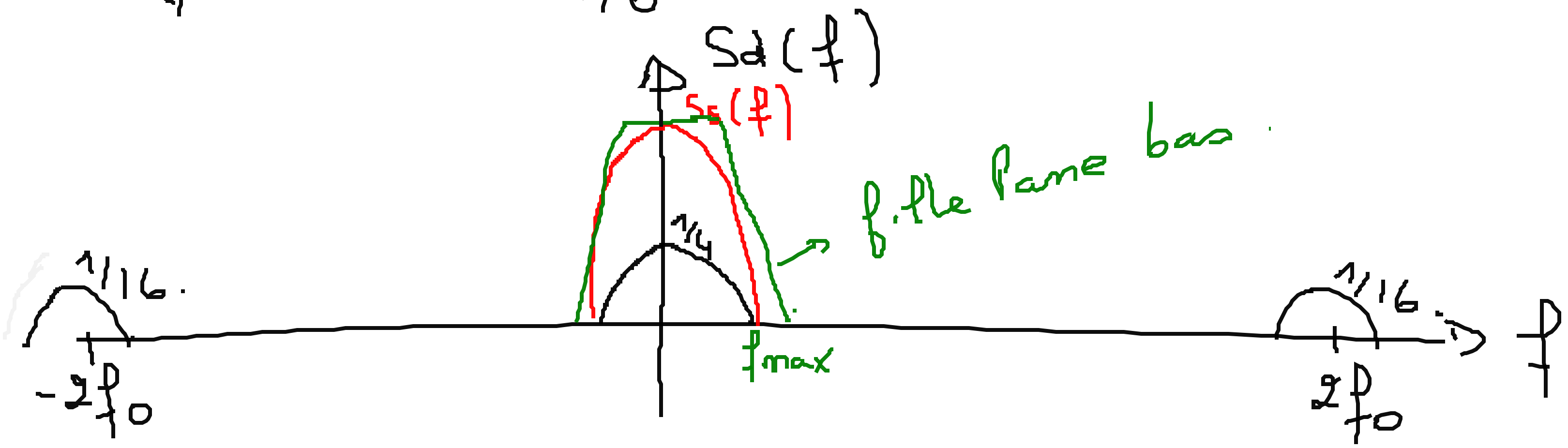
$$= TF\left(\frac{1}{4} C_s(\tau) + \frac{1}{8} C_s(\tau) \cos 4\pi f_0 \tau\right)$$

$2 \times 2 f_0$

$$= \frac{1}{4} S_s(f) + \frac{1}{8} S_s(f) * TF(\cos 4\pi f_0 \tau)$$

$$\frac{1}{2} \delta(f - 2f_0) + \frac{1}{2} \delta(f + 2f_0)$$

$$= \frac{1}{4} S_s(f) + \frac{1}{16} S_s(f - 2f_0) + \frac{1}{16} S_s(f + 2f_0)$$



Pour récupérer $s(t)$ ⇒ filtre passe bas $f_c = f_{max}$.

11.1

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$h(t) = \delta(t) + a\,\delta(t - t_0)$$

$$x(t) = C \cdot \sin(2\pi f_0 t + \Phi_{(\omega)}) \qquad \Phi \in [0, 2\pi[$$

(1) $x(t)$ stationnaire au 2^{nd} ordre.

(2) $\cdot$ $E(x(t)) = C \cdot E(\sin 2\pi f_0 t + \Phi)$

$$= C \int_0^{2\pi} \sin(2\pi f_0 t + \Phi) \cdot \frac{1}{2\pi}\, d\Phi \qquad p_\Phi$$

$$= \frac{C}{2\pi} \left[\cos(2\pi f_0 t + \Phi)\right]_0^{2\pi} = 0$$

$\bullet$ $C_x(t, t-\tau) = C^2 \cdot E\left(\sin(2\pi f_0 t + \Phi) \cdot \sin 2\pi f_0 (t-\tau)\right)$

$$= \frac{C^2}{2} \cos 2\pi f_0 \tau \cdot$$ indépendante de t. Φ.

$$= C_x(\tau)$$

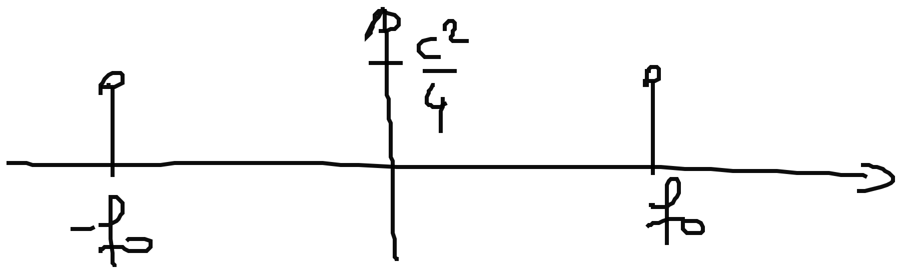
$\bullet$ $P_{moy} = C_x(0) = \frac{C^2}{2} < \infty$ finie

$X(t)$ Processus stationnaire au 2^- ordre.

③ $S_x(f) = TF(C_x(\tau))$

$= \frac{C^2}{2} TF(\cos 2\pi f_0 \tau)$

$= \frac{C^2}{4}\left(\delta(f-f_0) + \delta(f+f_0)\right)$



Remarque : $P_x = \int S_x(f)\,df = \frac{C^2}{4} + \frac{C^2}{4} = \frac{C^2}{2} = C_x(0)$

Etude de y(t)

y(t) est stationnaire au 2nd ordre

car $y(t) = h(t) * x(t)$

or x(t) est stationnaire au 2nd ordre

et h(t) est linéaire et stable.

$y(t) = (\delta(t) + a\delta(t-t_0)) * x(t)$

$= x(t) + a\,x(t-t_0)$

<u>objectif</u> calcul de $S_y(f)$

b) 1^e^ méthode : on calcule $C_y(\tau) \xrightarrow{TF} S_y(f)$

$$C_y(t, t-\tau) = E\left(y(t)\, y(t-\tau)\right)$$

$$= E\left(\left(x(t) + a\, x(t-t_0)\right)\left(x(t-\tau) + a\, x(t-t_0-\tau)\right)\right)$$

$$= E\left(\underbrace{x(t)\, x(t-\tau)}_{C_x(\tau)}\right) + E\left(\underbrace{a\, x(t-t_0)\, x(t-\tau)}_{a\, C_x(t_0-\tau)}\right)$$

$$+ E\left(\underbrace{a\, x(t)\, x(\overbrace{t-t_0-\tau}^{t-(t_0+\tau)})}_{C_x(t_0+\tau)}\right) + E\left(a^2\, \underbrace{x(t-t_0)\, x(t-t_0-\tau)}_{x(t)\, x(t-\tau) : C_x(\tau)}\right)$$

- $x(t)$ stat au 2^e^ ordre $C_x(t, t-\tau) = C_x(\tau)$

indépend de l'inst d'observation t.

$$x(t-t_0)\, x(t-\tau) = x(t')\, x(t'+t_0-\tau)$$

$t' = t - t_0$ $t = t' + t_0$ $\Rightarrow C_x(t_0-\tau)$

$$C_y(t, t-\tau) = C_y(\tau) = C_x(\tau) + a\, C_x(t_0-\tau)$$

$$+ a\, C_x(t_0+\tau) + a^2\, C_x(\tau)$$

$$= \frac{C^2}{2}\cos 2\pi f_0 \tau + a\frac{C^2}{2}\cos 2\pi f_0(t_0-\tau) + a\frac{C^2}{2}\cos 2\pi f_0(t_0+\tau)$$

$$+ a^2 \frac{C^2}{2}\cos 2\pi f_0 \tau.$$

$$= \frac{c^2}{2}(1+a^2)\cos 2\pi f_0 \tau + \frac{a c^2}{2} 2\cos 2\pi f_0 \tau_0 \cos 2\pi f_0 \tau$$

$$\boxed{C_y(\tau) = \frac{c^2}{2}\cos 2\pi f_0 \tau \left(1+a^2+2a\cos 2\pi f_0 \tau_0\right)}$$

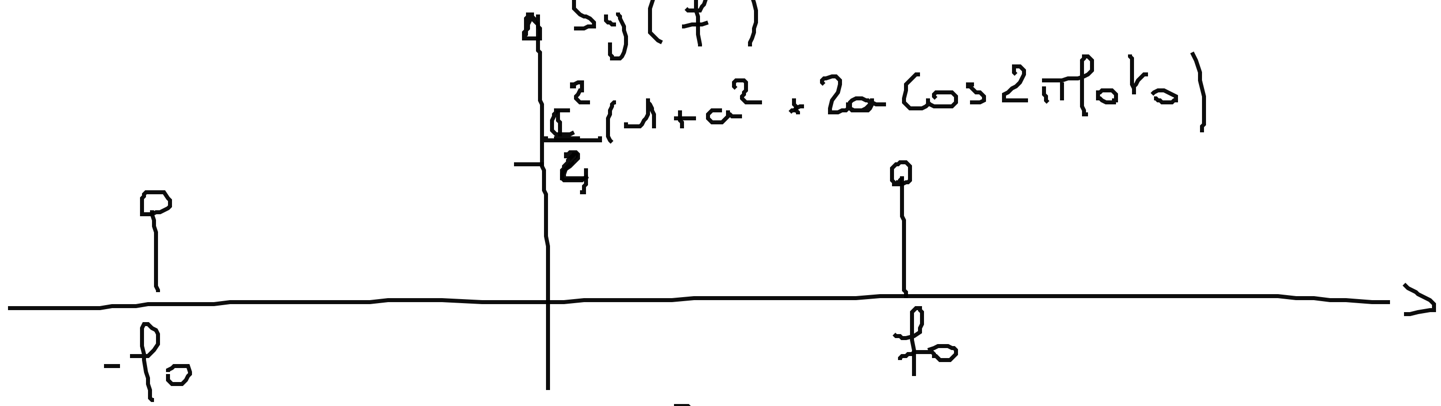
$\cos(a-b) + \cos(a+b)$
$= 2\cos a \cos b.$

$$S_y(f) = TF(C_y(\tau))$$

cste independ de τ

$$= \frac{c^2}{2} TF(\cos 2\pi f_0 \tau) * TF(1+a^2+2a\cos 2\pi f_0 \tau_0)$$

$$= \frac{c^2}{2}\left(\frac{1}{2}\delta(f-f_0)+\frac{1}{2}\delta(f+f_0)\right)\cdot\left(1+a^2+2a\cos 2\pi f_0 \tau_0\right)$$



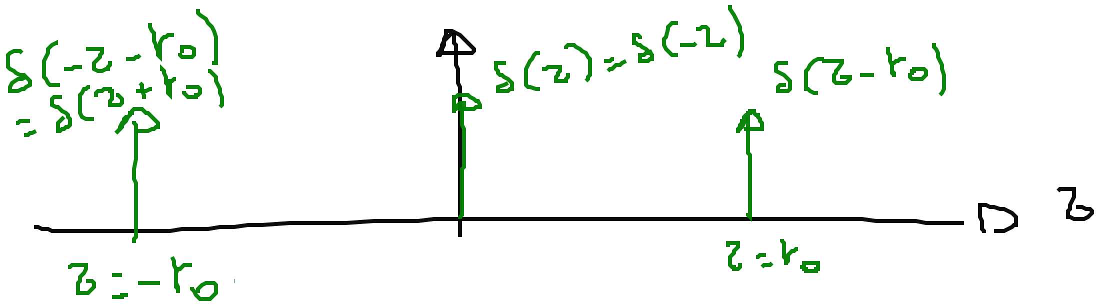
$$P_y = \int S_y(f)\,df = \frac{c^2}{2}(1+a^2+2a\cos 2\pi f_0 \tau_0)$$

$$= C_y(0) < \infty$$

2 méthode : Formule de filtrage si $h(\tau)$ compl

$$C_y(\tau) = C_x(\tau) * h(\tau) * h^*(-\tau)$$

$$C_y(\tau) = \left(\frac{c^2}{2}\cos 2\pi f_0 \tau\right) * \left(\delta(\tau) + a\,\delta(\tau - \tau_0)\right) * \left(\underbrace{\delta(-\tau)}_{=\delta(\tau)} + a\,\underbrace{\delta(-\tau-\tau_0)}_{\delta(\tau+\tau_0)}\right)$$



$$= \frac{c^2}{2}\cos 2\pi f_0 \tau * \left(\delta(\tau) + a\,\delta(\tau+\tau_0) + a\,\delta(\tau - \tau_0) + a^2\,\underbrace{\delta(\tau)}_{\delta(\tau-\tau_0+\tau_0)}\right)$$

$$= \frac{c^2}{2}\cos 2\pi f_0 \tau + a\frac{c^2}{2}\cos\left(2\pi f_0(\tau+\tau_0)\right) + a\frac{c^2}{2}\cos\left(2\pi f_0(\tau-\tau_0)\right) + \frac{a^2}{2}c^2\cos 2\pi f_0 \tau$$

$$= \frac{c^2}{2}\cos 2\pi f_0 \tau \left(1 + a^2 + 2a\cos 2\pi f_0 \tau_0\right)$$

Résultat idem à celui obtenu à 7.8)

→ $S_y(f)$ cf 8)

3° méthode : Domaine fréquentiel

$$S_y(f) = TF(C_y(\tau)) = TF(C_x(\tau) * h(\tau) * h^*(-\tau))$$

$$= S_x(f) \cdot H(f) \cdot H^*(f) = S_x(f) \cdot |H(f)|^2$$

$$H(f) = TF(h(\tau)) = TF(\delta(\tau) + a\,\delta(t - \tau_0))$$

$$= 1 + a\,e^{-2\pi j f \tau_0} = 1 + a\cos 2\pi f_0 \tau_0 - j\,a \sin 2\pi f_0 \tau_0$$

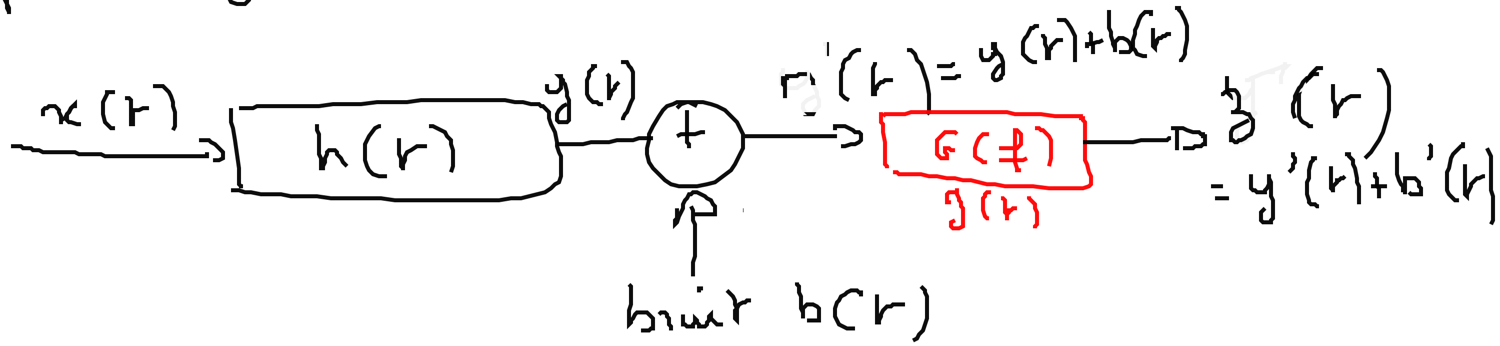
$$|H(f)|^2 = (1 + a\cos 2\pi f_0\tau_0)^2 + a^2 \sin^2 2\pi f_0 \tau_0$$

$$= 1 + 2a\cos 2\pi f_0\tau_0 + \underbrace{a^2\cos^2 2\pi f_0\tau_0 + a^2 \sin^2 2\pi f_0\tau_0}_{a^2}$$

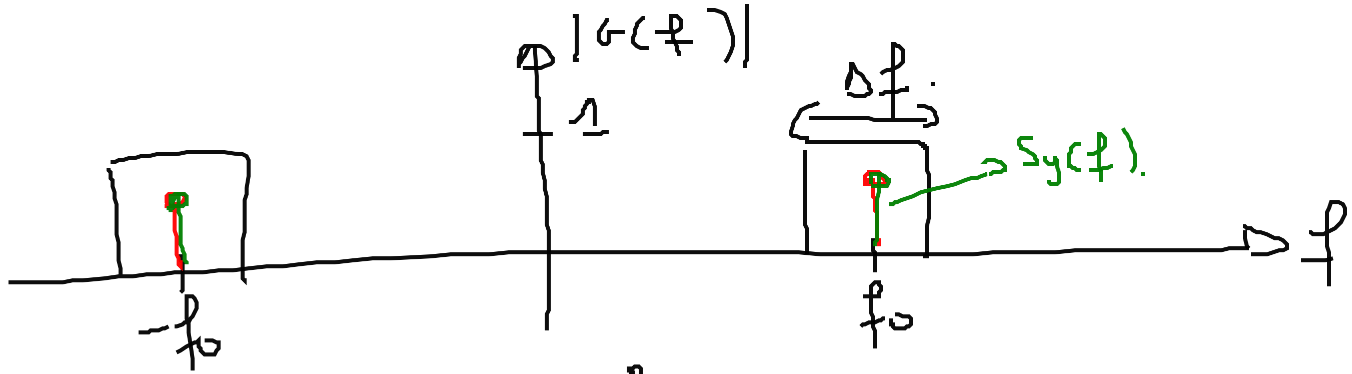
$$= 1 + a^2 + 2a\cos 2\pi f_0 \tau_0$$

$$\boxed{S_y(f) = \frac{c^2}{4}\left(\delta(f + f_0) + \delta(f - f_0)\right) \cdot 1 + a^2 + 2a\cos 2\pi f_0\tau_0}$$

Reception $y(t)$



Bruit : $S_b(f) = \frac{N_0}{2} \quad \forall f$.



$$S_z(f) = |G(f)|^2 \cdot S_r(f)$$

On s'intéresse au signal utile

$$S_{y'}(f) = |G(f)|^2 \cdot S_y(f) \qquad \text{si } \Delta f \neq 0$$

$$= |G(f)|^2 \cdot \frac{c^2}{2}\left(\delta(f-f_0) + \delta(f+f_0)\right) * \left(1+a^2+\dots\right)$$

$$|G(f_0)|^2 = 1$$

$$S_{y'}(f) = S_y(f)$$

Pour le bruit

$$S_{b'}(f) = |G(f)|^2 \cdot S_b(f)$$

$$= \frac{N_0}{2} \cdot \left(\mathrm{rect}\left(\frac{f-f_0}{\Delta f}\right) + \mathrm{rect}\left(\frac{f+f_0}{\Delta f}\right)\right)$$

$\Rightarrow$ Rapport $\dfrac{S}{N} = \dfrac{\text{Puissance du Signal en sortie de } G(f)}{\text{Puissance du bruit en sortie de } G(f)}$

$$P_{bruit} = \int S_{b'}(f)\, df = \frac{N_0}{2} \int_{-f_0-\frac{\Delta f}{2}}^{-f_0+\Delta f/2} 1 \, df + \frac{N_0}{2} \int_{f_0-\Delta f/2}^{f_0+\Delta f/2} 1 \, df.$$

$$= \frac{N_0}{2} \cdot \Delta f + \frac{N_0}{2} \cdot \Delta f = N_0 \Delta f.$$

$$P_{y'} = \int S_{y'}(f)\, df = \int S_y(f)\, df = \frac{C^2}{2} \left(1 + a^2 + 2a \cos 2\pi f_0 t_0\right)$$

$$\Rightarrow \frac{S}{N} = \frac{C^2}{2} \frac{\left(1 + a^2 + 2a \cos 2\pi f_0 t_0\right)}{N_0 \Delta f}.$$

$\frac{S}{N} \nearrow$ si $\Delta f \searrow$