$$E(X) = \int_{-\infty}^{+\infty} x \cdot f_X(x)\,dx$$

$$= \int_{-\infty}^{0} x \times 0\,dx + \int_{0}^{+\infty} x \cdot \lambda e^{-\lambda x}\,dx$$

$$= \int_{0}^{+\infty} \underbrace{x}_{u} \cdot \underbrace{\lambda e^{-\lambda x}}_{v'}\,dx.$$

Intégration par parties

$u(x) = x \qquad v'(x) = \lambda e^{-\lambda x}$

$u'(x) = 1 \qquad v(x) = -e^{-\lambda x}$

$$E(X) = \Big[\underbrace{x}_{u} \cdot (\underbrace{-e^{-\lambda x}}_{v})\Big]_{0}^{+\infty} - \int_{0}^{+\infty} \underbrace{1}_{u'} \cdot (\underbrace{-e^{-\lambda x}}_{v})\,dx$$

$$= \underbrace{0}_{\text{Thm des croissances comparées}} + \int_{0}^{+\infty} e^{-\lambda x}\,dx$$

$$= \left[-\frac{e^{-\lambda x}}{\lambda}\right]_{0}^{+\infty} = 0 - \left(-\frac{e^{0}}{\lambda}\right) = \frac{1}{\lambda}.$$