

$$E(X) = \int_{-\infty}^{+\infty} x \cdot f(x) dx$$

$$= \int_{-\infty}^0 x \times 0 dx + \int_0^{+\infty} x \cdot \lambda e^{-\lambda x} dx$$

$$= \int_0^{+\infty} \underbrace{x}_{u} \cdot \underbrace{\lambda e^{-\lambda x}}_{v'} dx$$

Intégration par parties

$$\begin{aligned} u(x) &= x & v'(x) &= \lambda e^{-\lambda x} \\ u'(x) &= 1 & v(x) &= -e^{-\lambda x} \end{aligned}$$

$$\begin{aligned} E(X) &= \left[ \underbrace{x}_{u} \cdot \underbrace{(-e^{-\lambda x})}_{v} \right]_0^{+\infty} - \int_0^{+\infty} \underbrace{1}_{u'} \cdot \underbrace{(-\lambda e^{-\lambda x})}_{v'} dx \\ &= \underbrace{0}_{\text{lim des croissances comparées}} + \int_0^{+\infty} \lambda e^{-\lambda x} dx \end{aligned}$$

$$= \left[ -\frac{e^{-\lambda x}}{\lambda} \right]_0^{+\infty} = 0 - \left( -\frac{e^0}{\lambda} \right) = \frac{1}{\lambda}$$