

$$\left\{ \begin{array}{l} \text{Si } X \sim \mathcal{N}(m_1; \sigma_1^2), Y \sim \mathcal{N}(m_2; \sigma_2^2), X \text{ e } Y \text{ indep.} \\ \text{alors } Z = aX + bY \sim \mathcal{N}(m; \sigma) \end{array} \right.$$

$$m: E(Z) = E(aX + bY) = aE(X) + bE(Y) = am_1 + bm_2$$

$$\sigma^2 = \text{Var } Z = \text{Var}(aX + bY) = \text{Var}(aX) + \text{Var}(bY)$$

$$= a^2 \text{Var } X + b^2 \text{Var } Y$$

$$= a^2 \sigma_1^2 + b^2 \sigma_2^2$$

$$\sigma = \sqrt{a^2 \sigma_1^2 + b^2 \sigma_2^2}$$

$$\begin{array}{l} E(X+Y) = E(X) + E(Y) \\ \text{Var}(X+Y) = \text{Var } X + \text{Var } Y \end{array}$$

$$\begin{array}{l} E(aX) = aE(X) \\ \text{Var}(aX) = a^2 \text{Var } X \end{array}$$

$$E((aX)^2) = \int_{-\infty}^{+\infty} \underbrace{a^2 x^2}_{= a^2} f(x) dx = a^2 \int_{-\infty}^{+\infty} x^2 f(x) dx$$

$$\begin{array}{l} X \sim \mathcal{N}(5, 2^2) \\ Y \sim \mathcal{N}(3, 4^2) \\ X \text{ e } Y \text{ indep.} \end{array}$$

$$Z = 2X - Y$$

$$E(Z) = 2E(X) - E(Y) = 2 \times 5 - 3 = 7$$

$$\text{Var } Z = 4 \text{Var } X + \text{Var } Y = 4 \times 2^2 + 4^2 = 24$$

$$Z \sim \mathcal{N}(7, 24)$$