

$$\begin{aligned} \text{Var}(X) &= E[(X - E(X))^2] \\ &= \underline{E(X^2)} - \underbrace{E^2(X)}_{\mu_X^2} \end{aligned}$$

Théorème Transfert

$$E[g(x)] = \int \underbrace{g(x)}_{x^2} \underbrace{f_X(x)}_{x^2} dx$$

$$\begin{aligned} E[X^2] &= \int_{\mathbb{R}} x^2 f_X(x) dx = \int_0^{\theta} x^2 \frac{2x}{\theta^2} dx \\ &= \frac{2}{\theta^2} \int_0^{\theta} x^3 dx = \frac{2}{\theta^2} \left[\frac{x^4}{4} \right]_0^{\theta} \\ &= \frac{2}{\theta^2} \frac{\theta^4}{4} = \frac{\theta^2}{2} \end{aligned}$$

$$\begin{aligned} \Rightarrow \text{Var}(X) &= E(X^2) - \mu_X^2 = \frac{\theta^2}{2} - \left(\frac{2}{3}\theta\right)^2 \\ &= \theta^2 \left(\frac{1}{2} - \frac{4}{9}\right) = \frac{\theta^2}{18} \end{aligned}$$