

(2) Calculer $E(X)$, $\text{Var}(X)$

$$\begin{aligned} \mu_X = E[X] &= \int_{\mathbb{R}} x f_X(x) dx & f_X(x) &= \begin{cases} \frac{1}{b-a} & \text{si } x \in [a, b] \\ 0 & \text{ailleurs} \end{cases} \\ &= \int_a^b x \frac{1}{b-a} dx \\ &= \frac{1}{b-a} \left[\frac{x^2}{2} \right]_a^b = \frac{b^2 - a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} \\ &= \frac{a+b}{2} \end{aligned}$$

$$\text{Var}(X) = E[(X - \mu_X)^2] = \underbrace{E[X^2]} - \mu_X^2$$

$$\begin{aligned} E[X^2] &= \int_{\mathbb{R}} x^2 f_X(x) dx = \int_a^b \frac{x^2}{b-a} = \frac{1}{b-a} \left[\frac{x^3}{3} \right]_a^b \\ &= \frac{b^3 - a^3}{3(b-a)} = \frac{a^2 + ab + b^2}{3} \end{aligned}$$

$$\Rightarrow \text{Var}(X) = \frac{a^2 + ab + b^2}{3} - \left(\frac{a+b}{2} \right)^2 = \frac{(b-a)^2}{12}$$